Determinations of $|V_{ub}|$ with inclusive techniques at LEP

M. Battaglia[a]*

[a]Dept. of Physics, University of Helsinki,
FIN-00014 Helsinki, Finland

The charmless semileptonic decay B branching fraction has been measured, using inclusive techniques, by the ALEPH, DELPHI and L3 experiments at LEP. The average of their results is

$$\mathrm{BR}(b \to X_u \ell \bar{\nu}) = (1.74 \pm 0.37\ (\text{stat.+exp.}) \pm 0.38\ (b \to c) \pm 0.21\ (b \to u)) \times 10^{-3}.$$

From this result the value of the $|V_{ub}|$ element in the CKM mixing matrix has been derived, using OPE predictions, obtaining:

$$|V_{ub}| = (4.13^{+0.42}_{-0.47}\ (\text{stat.} + \text{det.})^{+0.43}_{-0.48}\ (\text{b} \to \text{c syst.})^{+0.24}_{-0.25}\ (\text{b} \to \text{u sys.}) \pm 0.02\ (\tau_b) \pm 0.20\ (\text{HQE})) \times 10^{-3}.$$

1. INTRODUCTION

The accuracy in the determination of the $|V_{ub}|$ element in the Cabibbo-Kobayashi-Maskawa mixing matrix [1] plays an important role in the study of the unitarity triangle and in the related tests of the Standard Model (SM). In particular, a non-vanishing $|V_{ub}|$ is essential to preserve the possibility to describe CP violation within the SM and its value places a direct constraint on the magnitude of the CP violating phase β.

The first determination of the magnitude of $|V_{ub}|$ was obtained from the yield of leptons produced with momentum above the kinematic limit for $b \to X_c \ell \bar{\nu}$ transitions, first reported by CLEO [2] and soon confirmed by ARGUS [3]. However this method is sensitive to only $\simeq 10\%$ of the inclusive charmless semileptonic (s.l.) yield, and the extraction of $|V_{ub}|$ is subject to a large model dependence.

More recently, exclusive $B \to \pi \ell \bar{\nu}$ and $B \to \rho \ell \bar{\nu}$ decays have been measured by CLEO [4]. The determination of $|V_{ub}|$ from exclusive s.l. decays has still a significant model dependence. First lattice estimates of the relevant form factors indicate that a significant reduction of these uncertainties may be expected in the future.

The extraction of $|V_{ub}|$ from the distribution of the invariant mass M_X of the hadronic system recoiling against the lepton pair peaked, for $b \to X_u \ell \bar{\nu}$, at a significantly lower value than for $b \to X_c \ell \bar{\nu}$ was proposed several years ago [5], and it has recently been the subject of new theoretical calculations [6,7]. If $b \to u$ transitions can be discriminated from the dominant $b \to c$ background up to $M_X \simeq M(D)$, this method is sensitive to $\simeq 80\%$ of the charmless s.l. B decay rate. Further, if no preferential weight is given to low mass states in the event selection, the non-perturbative effects are expected to be small and the OPE description of the transition has been shown to be accurate away from the resonance region.

The experimental challenge comes from the requirement to isolate the $b \to u$ contribution to the s.l. yield from the $\simeq 60$ times larger $b \to c$ one while ensuring a uniform sampling of the decay phase space to avoid biases towards a few exclusive low-mass, low-multiplicity states, such as $\pi \ell \bar{\nu}$ and $\rho \ell \bar{\nu}$. In principle, this method is well suited for the LEP experiments, where the recorded statistics is not sufficient for studying the exclusive decay modes with good accuracy while the significant boost of the B hadrons, the separation of b and $\bar{b}$ decay products in opposite hemispheres, and the good secondary vertex re-

---

*present address: CERN/EP Division, CH-1211 Geneva

construction capabilities make the study of inclusive decay B decays possible. The analysis techniques adopted by the ALEPH [8], DELPHI [9] and L3 [10] Collaborations are based on the observation that $b \to X_u \ell \bar{\nu}$ decays can be inclusively discriminated from $b \to X_c \ell \bar{\nu}$ by exploiting the differences in the invariant mass and kaon content of the secondary hadronic system, in the decay multiplicity and in the decay vertex topology. These features have been used differently in the three analyses, resulting in determinations of the charmless s.l. branching fraction obtained from samples with varying efficiency and purity and with systematic uncertainties that are only partially correlated.

2. EXPERIMENTAL RESULTS

The analyses consist of three main steps: i) inclusive selection of s.l. B decay candidates, ii) definition of a subsample enriched in $b \to u$ transitions and iii) measurement of the charmless s.l. branching fraction. All experiments started from a sample of events containing an identified electron or muon selected from a combined data set of $\simeq 8.2$ M hadronic Z^0 decays. ALEPH and DELPHI also imposed b-tag criteria to reject light quark and charm backgrounds. ALEPH used two neural networks (NN) to separate charged and neutral B decay products from fragmentation particles, reconstructed the secondary hadronic system and the B rest frame and computed twenty $b \to u$ discriminating kinematical variables in this frame. DELPHI also reconstructed the secondary hadronic system mass and the B rest frame, but using a particle likelihood variable and an iterative topological reconstruction procedure and estimated the lepton energy in the B rest frame E_ℓ^* for each event. L3 adopted a consecutive cut analysis based on the kinematics of the two most energetic hadrons in the same hemisphere as the tagged lepton. All the three experiments observed a significant excess of events with the characteristics expected for $b \to X_u \ell \bar{\nu}$ decays.

ALEPH combined the selected discriminating variables by means of another NN to obtain a global discriminant variable NN_{bu} (see Figure 1). The number of $b \to X_u \ell \bar{\nu}$ candidates in the data
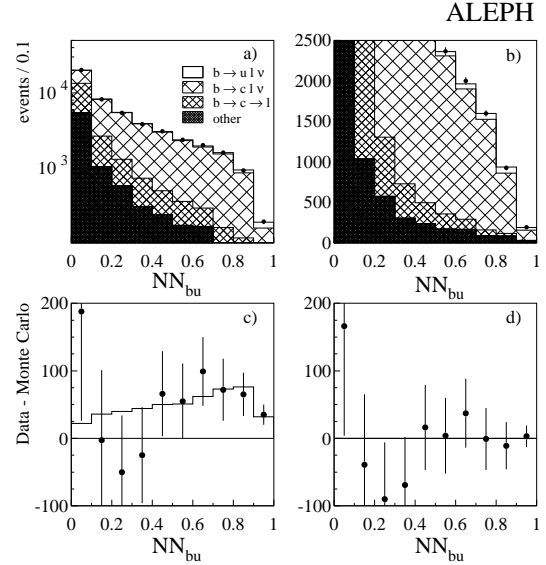

Figure 1. The $b \to u$ discriminating NN output for the ALEPH analysis. a) and c) include only the simulated backgrounds while b) and d) give the comparison with the data after including $b \to u$ s.l. decays corresponding to the fitted branching fraction

was extracted by a binned likelihood fit to the NN_{bu} and converted into the charmless s.l. BR. DELPHI divided the reconstructed events in four classes on the basis of the M_X value and of enrichment criteria based on the relative position of the lepton w.r.t. the secondary vertex and the presence of tagged kaons. Of these classes the $M_X < 1.6$ GeV/c^2 - $b \to u$ enriched one was expected to contain almost 70% of the s.l. $b \to u$ decays while the other classes, depleted of signal events, were used to monitor the background modeling in the simulation (see Figure 2). The ratio $|V_{ub}|/|V_{cb}|$ was extracted, together with the overall data/MC normalization, from the fraction of $b \to X_u \ell \bar{\nu}$ candidates observed in the data using a two parameter likelihood fit to the number of events in each of the four classes and to their E_ℓ^* distributions. Finally L3 extracted the charmless s.l. B branching fraction by counting the excess of events over the estimated background after having applied their selection criteria.
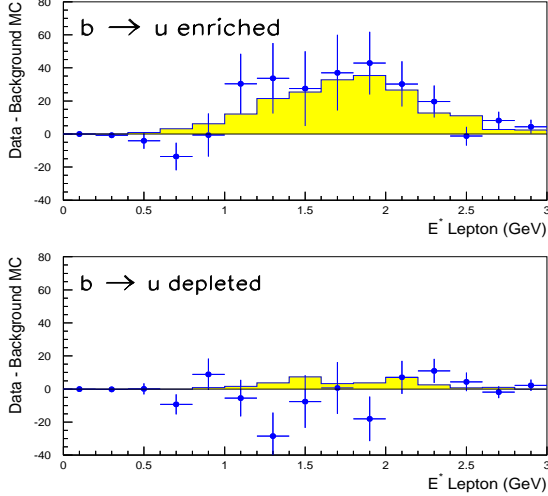
Starting from a natural signal-to-background

Figure 2. Background subtracted E_ℓ^* distributions for the DELPHI analysis: the $b \to u$ enriched decays with $M_X < 1.6$ GeV/c^2 (upper plot) and $b \to u$ depleted decays with $M_X < 1.6$ GeV/c^2 (lower plot). The shaded histograms show the expected E_ℓ^* distribution for signal $b \to u$ s.l. decays normalized to the amount of signal corresponding to the fitted $|V_{ub}|/|V_{cb}|$ value.
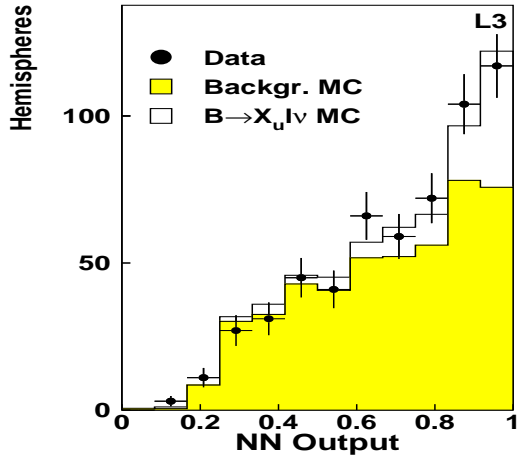

Figure 3. Output of a NN combining the discriminating variables of the L3 analysis. The amount of signal in the simulation has been scaled to the measured BR. This NN has only been used as consistency check.

ratio, S/B, of about 0.02, ALEPH obtained S/B $= 0.07$ with an efficiency $\epsilon = 11\%$, DELPHI S/B $= 0.10$ with $\epsilon = 6.5\%$ and L3 S/B $= 0.16$ with $\epsilon = 1.5\%$. The results are summarized in Table 1. Several consistency checks were performed by the three experiments. ALEPH explicitly used vertexing variables, DELPHI performed a search for fully reconstructed $B \to \pi\ell\bar{\nu}$, $\rho\ell\bar{\nu}$ decays and L3 validated the excess of events by constructing a $b \to u$ discriminating NN (see Figure 3).

3. AVERAGE BR($b \to X_u\ell\bar{\nu}$) AND EXTRACTION OF $|V_{ub}|$

The three measurements of BR($b \to X_u\ell\bar{\nu}$) have been averaged using the Best Linear Unbiased Estimate (B.L.U.E.) technique [11]. This technique provides with an unbiased estimate BR_{LEP} that is a linear combination of the different measurements BR_i corresponding to the minimum possible uncertainty σ:

$$BR_{LEP} = \frac{\sum_{i=1}^{3} \sum_{j=1}^{3} BR_i (E^{-1})_{ij}}{\sum_{i=1}^{3} \sum_{j=1}^{3} (E^{-1})_{ij}} \qquad (1)$$

with $\sigma^2 = \frac{1}{\sum_{i=1}^{3} \sum_{j=1}^{3} (E^{-1})_{ij}}$ where E is the error matrix including the off-diagonal terms giving the correlations between pairs of measurements.

The sources of correlated systematics belong to both the description of background $b \to c$ and to the modeling of signal $b \to u$ transitions. The differences in the analysis techniques adopted by the three experiments are reflected by differences in the sizes of the systematic uncertainties estimated from each common source. Important common systematics are due to the charm topological branching ratios and to the rate of $D \to K^0$ decays. ALEPH and L3 are also sensitive to the uncertainties in the b fragmentation function due to the use of kinematical variables for enriching in $b \to X_u\ell\bar{\nu}$. The DELPHI result depends on the assumed composition in b hadron species due to the use of kaon anti-tagging to reject $b \to c$, thus rejecting also B_s and Λ_b decays. The signal $b \to u$ systematics have been grouped in *inclusive model* and *exclusive model* and assumed to be fully correlated. The first corresponds to the uncertainty in modeling the kine-

Table 1
The results on $\mathrm{BR}(b \to X_u \ell \bar{\nu})$ from the LEP experiments with the sources of the statistical, experimental, model uncorrelated and model correlated uncertainties.

Experiment	$\mathrm{BR}(b \to X_u \ell \bar{\nu})$	(stat.)	(exp.)	(uncorrelated)	(correlated)
ALEPH [8]	(1.73	± 0.48	± 0.29	$\pm 0.29 \left(\genfrac{}{}{0pt}{}{\pm 0.29\ b\to c}{\pm 0.08\ b\to u}\right)$	$\pm 0.47 \left(\genfrac{}{}{0pt}{}{\pm 0.43\ b\to c}{\pm 0.19\ b\to u}\right)) \times 10^{-3}$
DELPHI [9]	(1.69	± 0.37	± 0.39	$\pm 0.18 \left(\genfrac{}{}{0pt}{}{\pm 0.13\ b\to c}{\pm 0.13\ b\to u}\right)$	$\pm 0.42 \left(\genfrac{}{}{0pt}{}{\pm 0.34\ b\to c}{\pm 0.20\ b\to u}\right)) \times 10^{-3}$
L3 [10]	(3.30	± 1.00	± 0.80	$\pm 0.68 \left(\genfrac{}{}{0pt}{}{\pm 0.68\ b\to c}{-\ b\to u}\right)$	$\pm 1.40 \left(\genfrac{}{}{0pt}{}{\pm 1.29\ b\to c}{\pm 0.54\ b\to u}\right)) \times 10^{-3}$

matics of the b-quark in the heavy hadron. It has been estimated from the spread of the results obtained with the ACCMM model [12], a shape function, describing the distribution of the light-cone residual momentum of the heavy quark inside the hadron [13–15] and the parton model [16] in the ALEPH and DELPHI analyses and from the uncertainties in the single π and the lepton energy spectra for L3. The *exclusive model* uncertainty arises from the modeling of the hadronic final state in the $b \to X_u \ell \bar{\nu}$ decay. These uncertainties have been estimated by replacing the parton shower fragmentation model in JETSET [17] with the fully exclusive ISGW2 [18] model by ALEPH and DELPHI and by propagating a 100% uncertainty on the $B \to \pi \ell \bar{\nu}$ rate by L3. Using the inputs from Table 1, the LEP average value for $\mathrm{BR}(b \to X_u \ell \bar{\nu})$ was found to be: $\mathrm{BR}(b \to X_u \ell \bar{\nu})$ $= (1.74 \pm 0.37\,(\text{stat.+exp.}) \pm 0.38\,(b \to c) \pm 0.21$ $(b \to u)) \times 10^{-3} = (1.74 \pm 0.57) \times 10^{-3}$ with a confidence level for the combination of 0.723 [19].

The value of the $|V_{ub}|$ element has been extracted by using the following relationship derived in the context of Heavy Quark Expansion [20,21]

$$|V_{ub}| = .00445\,\left(\frac{\mathrm{BR}(b \to X_u \ell \bar{\nu})}{0.002}\frac{1.55\mathrm{ps}}{\tau_b}\right)^{\frac{1}{2}} \times A \quad (2)$$

with $A = (1 \pm 0.020(\mathrm{QCD}) \pm 0.035(\mathrm{m_b}))$ where the value $m_b = (4.58 \pm 0.06)$ GeV/c^2 has been assumed [22]. The uncertainties have been convoluted together assuming them to be Gaussian in $\mathrm{BR}(b \to X_u \ell \bar{\nu})$, with the exception of the theoretical uncertainty on A assumed to
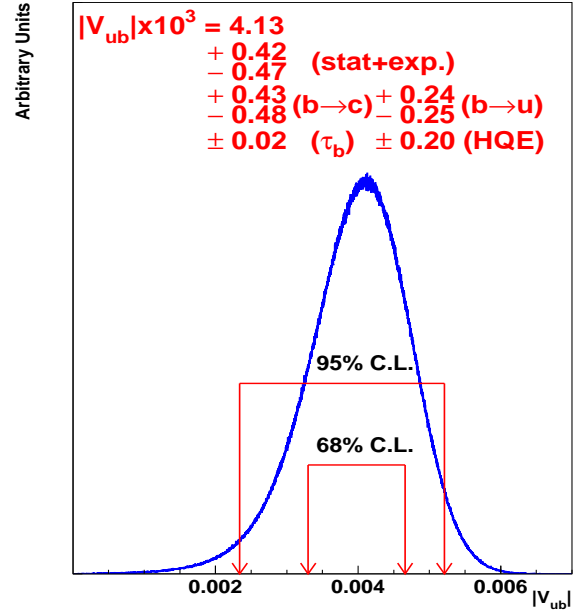

Figure 4. The probability density function for $|V_{ub}|$ corresponding to the LEP average value of $\mathrm{BR}(b \to X_u \ell \bar{\nu})$ with the intervals corresponding to the 68% C.L. and 95 % C.L. indicated.

be Gaussian in $|V_{ub}|$. The resulting probability density distribution is shown in Figure 4 and gives $|V_{ub}| = (4.13^{+0.63}_{-0.75}) \times 10^{-3}$ at 68% C.L. and $|V_{ub}| = (4.13^{+1.18}_{-1.71}) \times 10^{-3}$ at 95% C.L. The part of this function in the negative, unphysical region is negligible, corresponding to only 0.12%. By repeating this procedure separately for each

systematic, the detailed result for the 68% C.L. is:
$|V_{ub}| = (4.13^{+0.42}_{-0.47}(\text{stat.} + \text{det.})^{+0.43}_{-0.48}(\text{b} \to \text{c syst.})$
$^{+0.24}_{-0.25}(\text{b} \to \text{u sys.}) \pm 0.02(\tau_b) \pm 0.20(\text{HQE})) \times 10^{-3}$.

4. DISCUSSION AND CONCLUSIONS

The LEP analyses have demonstrated the feasibility of an inclusive determination of the charmless s.l. B branching fraction by discriminating $b \to u$ from $b \to c$ decays on the basis of the mass, multiplicity and kaon content of their secondary hadronic system and of their decay topology. Differently exploited by three of the LEP experiments, these event characteristics have been used to obtain clear signals for the decay, to measure its branching fraction and to derive a LEP combined value of $|V_{ub}| = (4.13^{+0.63}_{-0.75}) \times 10^{-3}$ at 68% C.L. with the relative uncertainty due to the $b \to u$ model below 10%. This result agrees with the recent CLEO determination [4] using the exclusive $B \to \rho\ell\bar{\nu}$ decay branching fraction giving $|V_{ub}| = (3.25^{+0.61}_{-0.64}) \times 10^{-3}$ where the uncertainty is dominated by a 17% model systematic mostly uncorrelated with that of the LEP measurement. The agreement between the inclusive and exclusive determinations, as in the case of $|V_{cb}|$, is encouraging as a test of the underlying theory assumptions and to control possible violations of quark-hadron duality in semileptonic B decays.

ACKNOWLEDGEMENTS

REFERENCES

1. N. Cabibbo, *Phys. Rev. Lett.* **10** (1963), 531 M. Kobayashi and K. Maskawa, *Prog. Theo. Phys.* **49** (1973), 652.
2. R. Fulton *et al.* (CLEO Collaboration), *Phys. Rev. Lett.* **64** (1990), 16.
3. H. Albrecht *et al.* (ARGUS Collaboration), *Phys. Lett.* **B 234** (1990), 409.
4. B.H. Behrens *et al.* (CLEO Collaboration), *Phys. Rev.* **D 61** (2000), 052001.
5. V. Barger, C.S. Kim and R.J.N. Phillips, *Phys. Lett.* **B 251** (1990), 629.
6. A.F. Falk, Z. Ligeti and M.B. Wise, *Phys. Lett.* **B 406** (1997), 225.
7. I. Bigi, R.D. Dikeman and N. Uraltsev, *Eur. Phys. J.* **C 4** (1998), 453.
8. R. Barate *et al.* (ALEPH Collaboration), *Eur. Phys. J.* **C6** (1999) 555.
9. P. Abreu *et al.* (DELPHI Collaboration), *Phys. Lett.* **B 478** (2000), 14.
10. M. Acciarri *et al.* (L3 Collaboration), *Phys. Lett.* **B 436** (1998) 174.
11. L. Lyons, D. Gibaut and P. Clifford, *Nucl. Instr. and Meth.* **A 270** (1988), 110.
12. G. Altarelli *et al.*, *Nucl. Phys.* **B 208** (1982), 365.
13. M. Neubert, *Phys. Rev.* **D 49** (1994), 4623 and *idem*, *Phys. Rev.* **D 50** (1994), 2037.
14. R. D. Dikeman, M. Shifman and N.G. Uraltsev, *Int. J. Mod. Phys.* **A 11** (1996), 571.
15. U. Aglietti and G. Ricciardi, CERN-TH/2000-071 and hep-ph/0003146.
16. A. Bareiss and E.A. Paschos, *Nucl. Phys.* **B 327** (1989), 353.
17. T. Sjostrand, *Comp. Phys. Comm.* **82** (1994), 74.
18. D. Skora and N. Isgur, *Phys. Rev.* **D 52** (1995), 2783.
19. D. Abbaneo *et al.* (The LEP V_{ub} WG), *Determination of the LEP Average BR(b $\to X_u\ell\bar{\nu}$) and Derivation of $|V_{ub}|$*, LEPVUB-00/01 and http://battagl.home.cern.ch/battagl/vub/
20. N. Uraltsev *et al.*, *Eur. Phys. J.* **C 4** (1998), 453 and *idem*, hep-ph/9905520.
21. A.H. Hoang, Z. Ligeti and A.V. Manohar, *Phys. Rev. Lett.* 82 (1999), 277 and *eadem*, *Phys. Rev.* D 59 (1999), 074017.
22. The ALEPH, CDF, DELPHI, L3, OPAL and SLD Collaborations, *Combined Results on b-hadron production rates, lifetimes, oscillations and semileptonic decays*, CERN-EP-2000-096.